The space density of spiral galaxies as function of their scale size, surface brightness and luminosity

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Abstract. The local space density of galaxies as a function of their basic structural parameters –luminosity, surface brightness and scale size– is still poorly known. Our poor knowledge is the result of strong selection biases against low surface brightness *and* small scale size galaxies in any optically selected sample. We derive bivariate space density distributions by correcting a sample of ∼1000 local Sb-Sdm spiral galaxies for its selection effects. We present a parameterization of these bivariate distributions, based on a Schechter type luminosity function and a log-normal scale size distribution at a given luminosity. We next calculate the bivariate distributions as function of redshift using the Hubble Deep Field, and conclude that at higher redshift there is a decrease in space density of luminous, large scale size galaxies, but the density of smaller galaxies stays nearly the same.

Keywords: spiral galaxies, selection effects, structural parameters

1. Introduction

In the last few decades many papers have been devoted to the determination of the luminosity function (LF), the central surface brightness distribution and, to some lesser extent, the scale size distribution of galaxies. These distribution determinations can actually not be separated due to the limits on the survey material on which these investigations are based. Any galaxy LF is only valid to the surface brightness limit of the survey. Any surface brightness distribution has validity limits depending on the scale size and/or magnitude limits of the survey. In this paper we investigate the galaxy bivariate distribution functions using combinations of luminosity, surface brightness and scale size.

Bivariate distribution functions have two important applications. First of all, using bivariate functions is the proper way to compare samples with different selection functions, especially when comparing samples at different redshifts. Secondly, bivariate distribution functions provide excellent checks for galaxy formation and evolution theories. Any complete galaxy formation theory will have to explain the bivariate distribution functions of structural galaxy parameters.

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2. Visibility Correction

Our aim is to determine the average space density of galaxies with certain properties in an average volume in the universe. Most field galaxy samples are not volume limited, but for instance magnitude or diameter limited. Not all galaxies have the same luminosity or diameter and therefore each galaxy will have a different distance range where it can be placed before dropping out of the sample due to the selection criteria. The maximum volume where a galaxy can be seen and included into the sample ($V_{\rm max}$) goes as the distance limits cubed, which results in galaxy samples being dominated by intrinsically bright and/or large galaxies, because they have a hugely larger visibility volume (Davies et al. 1994; McGaugh et al. 1995). We have to correct our sample for this selection effect to compute the true space density from the observed distribution.

In this paper we use one of the most simple selection effect correction methods available, the $V_{\rm max}$ correction method. Each galaxy gets a weight inverse proportional to its maximum visibility volume set by the selection limits. For a sample with upper ($D_{\rm max}$) and lower ($D_{\rm min}$) major axis diameter limits this leads to

$$V_{\rm max} = \frac{4\pi}{3} d^3 ((\frac{D_{\rm maj}}{D_{\rm max}})^3 - (\frac{D_{\rm maj}}{D_{\rm min}})^3) \qquad (1)$$

with d the distance and $D_{\rm maj}$ the major axis diameter of the galaxy. Other limits, like redshift, magnitude or sky fraction limits, that would limit $V_{\rm max}$, can trivially be taken into account as well. For a sample of galaxies which is complete to within the selection limits we can now define an estimator of the bivariate distribution function in parameters x and y: $\Phi(x,y) \approx \frac{1}{\Delta x \Delta y} \sum_i^N \frac{\delta^i}{V_{\rm max}^i}$, where i is summed over all N galaxies and $\delta^i = 1$ if (x_i,y_i) of a galaxy is in the $(x \pm \Delta x/2, y \pm \Delta y/2)$ bin range, otherwise 0.

The parameters used to select the galaxy sample can only be determined with finite accuracy, leading to what often is called the Malmquist edge-bias. Assuming a symmetric error distribution on the selection parameters (e.g. diameter/magnitude), objects have an equal chance of being scattered bins up as being scattered bins down. Because there are many more objects in the bins with smaller/fainter galaxies, on average more objects are scattered up than down and we will overestimate the number of objects in each bin. We correct for this Malmquist selection parameter uncertainty by calculating for each galaxy the weighted average $V_{\rm max}$ over the probability distribution of the selection parameters.

3. The nearby galaxy sample

We have used the sample described by Matthewson, Ford and Buchhorn (1992, MFB hereafter) as the starting point for our sample selection. With more than a thousand field galaxies it is large enough not to run immediately into low number statistics near the low surface brightness and/or small scale size selection borders (de Jong & Lacey 1999). The main drawback of the sample is its selection, as the sample was defined as a subsample from the ESO-Uppsala Galaxy Catalog, which is a catalog selected by eye from photographic plates.

The MFB sample is in essence a diameter limited sample of Sa-Sdm spiral galaxies and we can use Eq. (1) for the visibility correction. Our reselection from the ESO-Uppsala Catalog consists of 1003 galaxies, of which 860 having both MFB surface photometry and redshifts. The radial I-band luminosity profiles provided by MFB were used to calculate total magnitudes (M_I), half total light (effective) radii ($r_{\rm e}$) and the average surface brightnesses within these radii ($\langle\mu\rangle_{\rm e}$). The 1D luminosity profiles were decomposed in bulge and disk, using exponential light profiles for both disk and bulge (method described in de Jong 1996a), and disk only structural parameters were derived.

Distances were calculated from the redshifts using a Hubble constant of 65 $\mathrm{Mpc\,km^{-1}\,s}$, corrected for peculiar velocities for those galaxies included in the Mark III catalog (Willick et al. 1997). The Galactic foreground extinction corrections were calculated according to the precepts of Schlegel et al. (1998). We have used the internal extinction correction method introduced by Byun (1992, see also Giovanelli et al. 1995). A parameter for which the extinction correction has to be determined is first fitted against the maximum rotation velocity of the disk ($V_{\rm rot}$) to reduce the effect of distance dependent selection effects. The residuals on this fit are fitted against $\log(D_{\rm min}/D_{\rm maj})$ to empirically determine the extinction as function of inclination.

4. Space density distributions and a functional form

Using the corrections described in Sections 2&3, we calculate the space density of Sa-Sm galaxies in number of galaxies per Mpc^3 in the (M_I,$r_{\rm e}$)-plane as shown in Fig. 1. The accuracy of the distribution is limited on the small scale length, low luminosity end by selection effects (resulting in the large 95% confidence error bars), but these play hardly a rôle in determining the distribution on the other end of the diagram.

There is great interest in deriving a functional form to describe the observed bivariate distributions. Parameterizations of the bivariate

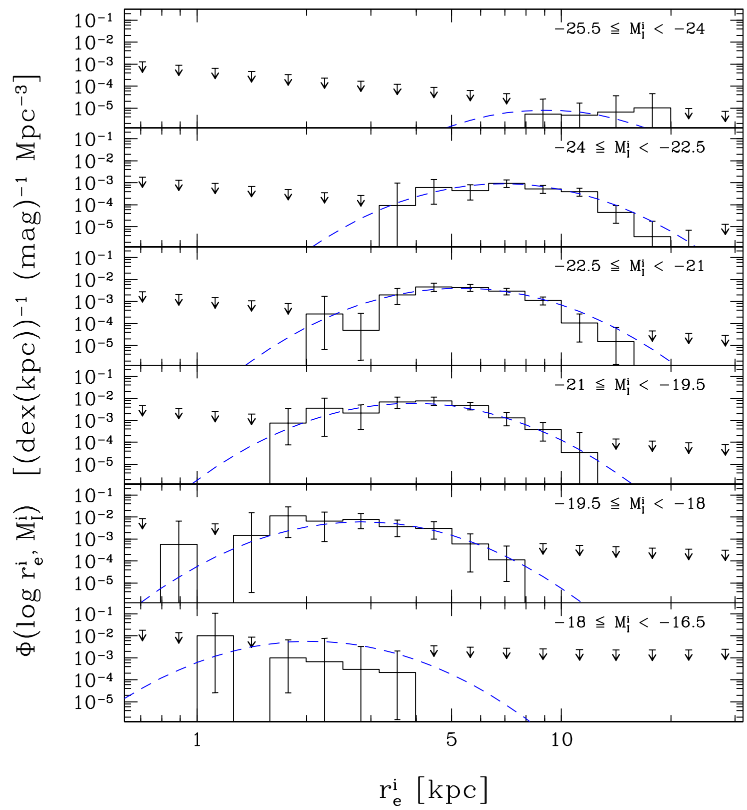

Figure 1. The density distribution of Sa-Sm galaxies as function of M_I versus r_e. The dashed line indicates the model described by Eq. (2)

distributions are useful when comparing distributions derived from differently selected samples and when studying galaxy redshift evolution. The parameterization can also be used in modeling where both galaxy luminosity and size are required (e.g. cross-section modeling of Lyman forest host galaxies). We will use here the bivariate distribution in the (M_I,r_e)-plane, as this distribution is most naturally connected to the galaxy formation scenario we are going to use to find a functional form for the bivariate distribution. Other parameter combinations can easily be derived using $M_I = \langle\mu\rangle_\mathrm{e} - 5\log(r_\mathrm{e}) - 2.5\log(2\pi)$.

To derive a parameterization of the bivariate distributions we will follow the Fall & Efstathiou (1980) disk galaxy formation theory. Galaxies form in this theory in hierarchically merging Dark Matter (DM) halos, giving rise to a distribution of DM halo masses described by the Press & Schechter (1974) theory, which formed the inspiration for the Schechter LF (1976). We will use a Schechter LF to describe the luminosity dimension of our bivariate distribution function.

In the Fall & Efstathiou (1980) model, the scale size of a galaxy is determined by its angular momentum, which is acquired by tidal torques from neighboring DM halos in the expanding universe. The total angular momentum of the system is usually expressed in terms of the dimensionless spin parameter $\lambda = J|E|^{1/2} M_{\rm tot}^{-5/2} G^{-1}$, with J the total angular momentum, E the total energy and $M_{\rm tot}$ the total mass of the system, all of which are dominated by the DM halo. N-body simulations (e.g. Warren et al. 1992) show that the distribution of λ values acquired from tidal torques can be approximated by a log-normal distribution with a dispersion $\sigma_\lambda \sim 0.5 - 0.7$.

A few simplifying approximations allow us to relate each of the factors in the spin equation to our observed bivariate distribution parameters. A perfect exponential disk of effective size $r_{\rm e}$, (baryonic) mass M_D, rotating with a flat rotation curve of velocity $V_{\rm rot}$ has $J_D \propto M_D r_{\rm e} V_{\rm rot}$. We assume that the specific angular momentum of the disk is equal to the specific angular momentum of the dark halo and therefore $J \propto M_D r_{\rm e} V_{\rm rot}$. From the virial theorem we get $E \propto V_{\rm rot}^2 M_{\rm tot}$. We use a power law between disk mass and luminosity: $M_D \propto L^\gamma$, with γ expected to be close to 1. The γ incorporates the effect of M_D/L variations due to gas mass fractions variations (de Blok & McGaugh 1997) and stellar populations variations (de Jong 1996b; Bell & de Jong 1999) which tend to be a function of L. We take the baryonic-to-DM ratio constant in each halo and assume that the same fraction of the baryonic mass always ends up in the disk, resulting in disk mass being proportional to total mass ($M_{\rm tot} \propto M_D$) in each halo. This is an assumption extremely hard to test, as nobody has seen the 'edge' of DM halos so far. Now we only need the Tully & Fisher relation ($L \propto V_{\rm rot}^\epsilon$, with $\epsilon \sim 3$ in the I-passband) to link the spin parameter λ to our observed bivariate distribution parameters.

These approximations yield $\lambda \propto r_{\rm e} L^{(2/\epsilon - \gamma)} \simeq r_{\rm e} L^{-1/3}$. As λ is expected to have a log-normal behavior, this means that, *at a given luminosity, we expect the distribution of scale sizes to be log-normal, and that the peak in the* $r_{\rm e}$ *distribution shifts with* $\sim L^{-1/3}$. Combining this result with the Schechter LF and using $\beta \equiv 2/\epsilon - \gamma$, the full bivariate function for space density as function of luminosity and effective radius becomes:

$$\begin{aligned}
&\Phi(\log r_{\rm e}, M)\, d\log r_{\rm e}\, dM = \qquad (2)\\
&\quad \frac{\Phi_0}{\sigma_\lambda \sqrt{2\pi}} \exp(-\frac{1}{2}\left[\frac{\log r_{\rm e}/r_{\rm e*} - 0.4(M - M_*)\beta}{\sigma_\lambda/\ln(10)}\right]^2)\\
&\quad 0.4\ln(10) 10^{-0.4(M-M_*)(\alpha+1)} \exp(-10^{-0.4(M-M_*)})\, d\log r_{\rm e}\, dM,
\end{aligned}$$

Table I. The results of fitting Eq. (2) to the observed distributions. Listed are the results for fits to the total galaxy parameters and to the disk only parameters, either with β fixed or free. The errors are 95% confidence limits. ϕ_0 is in mag^{-1} dex^{-1}(kpc) Mpc^{-3}, M_* in I-mag, r_{e*} in kpc and the other parameters are dimensionless

	Φ_0	α	M_*	r_{e*}	σ_λ	β
total	0.0031±0.0009	-1.05±0.12	-22.13±0.20	6.3±0.4	0.29±0.03	-1/3±0
disk	0.0033±0.0008	-1.04±0.11	-22.30±0.18	6.1±0.4	0.37±0.03	-1/3±0
total	0.0033±0.0008	-0.93±0.10	-22.17±0.17	6.1±0.4	0.28±0.02	-0.253±0.020
disk	0.0033±0.0007	-0.90±0.10	-22.38±0.16	5.9±0.3	0.36±0.03	-0.214±0.025

with the first line representing the log-normal scale size distribution and the second line the Schechter LF in magnitudes (M). In this equation Φ_0, α and M_* have the usual meaning in a Schechter LF.

Before we can fit Eq. (2) to the data, we will have to understand the uncertainty in the data points. The errors on the $V_{\rm max}$ corrected data points tend to be dominated by Poisson statistics. Especially in bins where we have few galaxies, these errors are highly asymmetric and we can not use a simple χ^2 minimalization routine to fit the data. The 95% confidence limits we plot on the histogram of Fig. 1 were calculated taking both the distance uncertainty and the Poisson confidence limits into account. In a similar fashion we can account for bins with no detections, calculating the Poisson probability distribution of a non-detection for a exponential disk with given bin parameters and these are indicated as upper limits in Fig. 1.

We used maximum likelihood fitting to determine the parameters in the bivariate distribution function. We used only the Poisson error distribution to calculate the likelihood distribution on each bin, which was minimized in the negative log. We used bootstrap resampling to estimate the errors on the bivariate distribution function parameters. Table I lists the fit results for two cases, one for M_I and $r_{\rm e}$ determined for the full galaxy (including bulge) and one for the disk only. Each case was fitted both with β fixed to $-1/3$ and β free. Surprisingly, the σ_λ values of $0.29-0.37$ are rather smaller than what is typically found for N-body cosmological simulations.

5. The Hubble Deep Field

With the Hubble space telescope we can now derive structural parameters of high redshift galaxies. Using the HDF (Williams 1996) we can

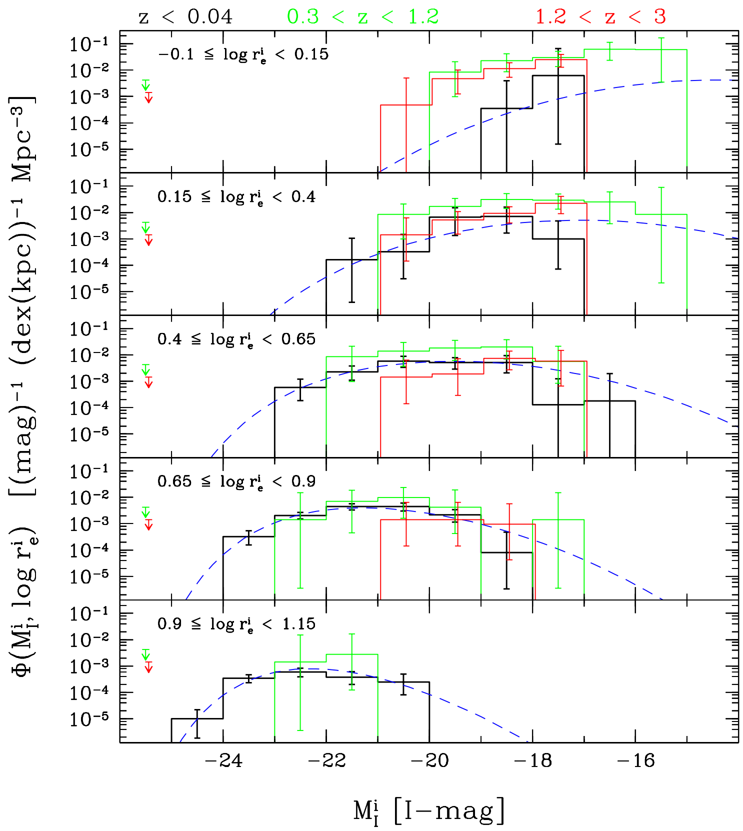

Figure 2. Redshift evolution of the bivariate density distribution of M_I versus r_e. The thick solid line $z < 0.04$, light gray line $0.3 < z < 1.2$ and dark gray $1.2 < z < 3$. The dashed line indicates the model described by Eq. (2)

estimate the bivariate space density of galaxies as function of redshift using similar techniques as described before. The situation is slightly more complicated, because we have to take into account cosmological corrections on the selection and derived parameters, most notable the $(1 + z)^4$ surface brightness dimming and the non-linear decrease in diameter with redshift. Furthermore we have to take bandpass shift into account with K-corrections. The derived $V_{\rm max}$ of a galaxy depends therefore on the assumed cosmology and galaxy evolution model.

We have used the structural galaxy parameters of Marleau & Simard (1998) and the photometric redshifts of Fernández-Soto et al. (1999) to derive to bivariate density distributions as function of redshift from the HDF. Even though limited by low number statistics, the result in Fig. 2 suggests that there is only moderate density evolution out to $z \sim 1.2$, but that the number density of bright, large scale size galaxies decreases at higher redshifts. The increase in space density of

the smallest galaxies ($\log r_{\rm e} < 0.15$) at the higher redshifts is probably due to the exclusion of Sm/Irr galaxies in the local sample, which is not the case for the HDF sample. This effect is therefore probably not real or only true for late-type galaxies.

6. Conclusions

We have quantified the local bivariate distribution function of spiral galaxies. It is well described by an Schechter LF in the luminosity dimension and a log-normal distribution shifting by $L^{-\beta}$ in the scale size dimension. This parameterization gives an accurate representation of the observed bivariate distributions, independently of whether one believes in hierarchical galaxy formation models or in CDM-like universes. A detailed analysis of galaxy formation in CDM-like universes paying attention to bivariate space density distributions will appear in Cole et al. (1999).

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